Introduction of intermediate defense measure in an evolutionary vaccination game

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Abstract

We establish a new mathematical framework of vaccination game that successfully can reproduce individual's decision making process by introducing couple of imperfect provisions besides taking the two extreme options of committing or not committing vaccination as their preliminary choices. A theoretical approach has been developed to model the real situation when intermediate defense measure (IDM) is introduced in an infinite and well-mixed population. Our theoretical result reveals that the introduction of IDM seems beneficial that can be justified in terms of the vaccination coverage, social average payoff, and other cost effect point of view during an epidemic outbreak.

1. Introduction

Over the last couple of decades, the severity of infectious disease spreading has been marked as one of the biggest threats imposed on human societies. From that viewpoint, pre-emptive vaccination could be the most effective intervention method for preventing disease transmission as well as reducing morbidity and mortality [1-2]. Evidence shows that there exists a complex interaction between human vaccinating behaviors and disease prevalence during an epidemic. Based on the outcomes obtained from various strategies, people can choose their best one to maximize their own payoffs which is considered as the fundamental premise of game theory. However, individuals may change their behavior in accordance to the changes in their perceived risk of becoming infected, and other socio-economic aspects. Therefore, evolutionary game theory (EGT) has been applied in the field of epidemiology by many researchers [3].

Voluntary vaccination scheme has a social dilemma structure that attracts people to free ride on other people's cooperative behavior rather getting themselves vaccinated. A vaccinator bears vaccination cost, time loss, and possible side effects entailed with vaccination while a non-vaccinator spends nothing but always undertakes the risk of being infected. In this context, it is indispensable to build up a prognostic theoretical model, such as Susceptible-Infected-Recovered (SIR) model incorporated with the EGT taking into account of individuals' decision making process toward vaccination or any other preventive measures popularly known as "vaccination game" [3-8]. Out of a bunch of studies made before, one pioneer work is proposed by Fu *et al.* who provided the original template of vaccination game [4]. After that Fukuda *et al*. developed a new strategy updating rule [2] and subsequently explored what happens if an epidemic spreading topology is inconsistent with the strategy-updating network [9], and if stubborn vaccinators and non-vaccinators are present in the social network [10]. Although most of the previous studies dealing with vaccination game presumed vaccination as a perfect provision but in the real world vaccination always does not work perfectly for diseases like Flu, Measles, HIV, and Influenza which gives rise to the concept of "effectiveness of vaccination" [11]. Furthermore, there are also some alternative protective measures, i.e. washing hands, wearing masks, gargling, and drinking energy drinks, termed as intermediate defense measure (IDM). Therefore, imperfect vaccination should be taken into account with significant impact on game theoretic analysis which reveals the fact that some vaccinated individuals obtain perfect immunity with a probability e, meanwhile, the remaining individuals fail to get immunity with a probability 1-e [11-13]. Generally, the efficiency of some protective measures other than vaccination can also be expressed probabilistically which ensures partial protection against infection spreading while costing less than vaccination.

Motivated by the aforementioned facts, we propose a simple compartment model (e.g. SIR/VMB) taking on a theoretical approach. Besides taking IDM as the third strategy, our study substantially addresses the fourth strategy that allows taking both vaccination and IDM at the same time during a single epidemic season. It also premises that some people might also like to invest for two preventive provisions at the same time anyway to avoid being infected if an IDM requires smaller cost vis-à-vis vaccination. It might also be justified from the seriousness of the disease spreading. Intuitively, during the severe spreading of an infectious disease, "fourth strategy" meaning taking both provisions is more reliable to deal with the worst scenario of epidemic.

The remainder of this paper is comprised as follows: Section 2 provides the model description in detail. Following to that, section 3 presents numerical results and the relevant discussions. At the very last section, this paper also elucidates a holistic summary of our findings that ends with the conclusion we draw.

2. Model description and results

2.1 Epidemic model

At the beginning, we consider SIR model which is then extended to establish SIR/VMB dynamics by considering vaccinators, third and fourth strategy holders as SIR variants. This modified SIR model allows three imperfect defense provisions considering the population being infinite and impeccably well mixed. Taking the model proposed by Kuga *et al*. as a guideline [11], we introduce two imperfect provisions (e.g. third strategy (denoted by M, explained later) and fourth strategy (denoted by B)) besides taking vaccination. Let us consider the effectiveness of vaccine be e ($0 \le e \le 1$) and the efficiency of IDM be η ($0 \le \eta \le 1$), meaning how taking the third strategy can reduce the risk of being infected. As a general assumption, we consider the cost of IDM must always be less than that of vaccination. Using a compartment model, the epidemic spreading dynamics is described where the individuals in a population can be classified into susceptible (S), infected (I), recovered (R), vaccinated (V), self-protected; meaning taking IDM (M) and combined protection; meaning taking both; vaccination and IDM (B) states. A non-provisioned susceptible individual may turn into infected if he/ she is exposed to infectious individuals at the disease transmission rate β (per day per person). An individual prepared with the defense against contagion who is in (S) may still have the possibility of becoming infected at the rate; $(1-\eta)\beta$. Meanwhile, an infected individual recovers at the recovery rate γ (per day) and the ratio of disease spreading rate with recovery rate (i.e. β/γ) is termed as basic reproduction number, denoted with R_0.

Table 1. Fractions of eight types of individuals.

Strategy/State	Healthy	Infected
Vaccinated	$HV(x,y,z,\infty)$	$IV(x,y,z,\infty)$
IDM	$HM(x,y,z,\infty)$	$IM(x,y,z,\infty)$
Vaccinated & IDM	$HB(x,y,z,\infty)$	$IB(x,y,z,\infty)$
Defector	$SFR(x,y,z,\infty)$	$FFR(x,y,z,\infty)$

Where,

HV: Healthy Vaccinator — IV: Infected Vaccinator
HM: Healthy IDM — IM: Infected IDM
HB: Healthy Both — IB: Infected Both
SFR: Successful free-rider — FFR: Failed free-rider

Table 2 Estimated payoff at the end of each season

Strategy/State	Healthy (H)	Infected (I)
Vaccinated (V)	$-C_{rv}$ (HV)	$-C_{rv}-1$ (IV)
IDM (M)	$-C_{rm}$ (HM)	$-C_{rm}-1$ (IM)
Vaccinated & IDM (B)	$-C_{rb}$ (HB)	$-C_{rb}-1$ (IM)
Defector (FR)	0 (SFR)	-1 (FFR)

The proposed model is governed by the following set of coupled differential equations:

$$\begin{aligned}
\frac{dS(x,y,z,t)}{dt} &= -\beta S(x,y,z,t)I(x,y,z,t),\\
\frac{dV(x,y,z,t)}{dt} &= -\beta\big(V(x,y,z,t)-eV(x,y,z,0)\big)I(x,y,z,t),\\
\frac{dM(x,y,z,t)}{dt} &= -(1-\eta)\beta M(x,y,z,t)I(x,y,z,t), \qquad (1)\\
\frac{dB(x,y,z,t)}{dt} &= -(1-\eta)\beta\big(B(x,y,z,t)-eB(x,y,z,0)\big)I(x,y,z,t),\\
\frac{dI(x,y,z,t)}{dt} &= \beta S(x,y,z,t)I(x,y,z,t)+\beta\big(V(x,y,z,t)-\\
&\quad eV(x,y,z,0)\big)I(x,y,z,t)+(1-\eta)\beta M(x,y,z,t)I(x,y,z,t)\\
&\quad +(1-\eta)\beta\big(B(x,y,z,t)-eB(x,y,z,0)\big)I(x,y,z,t)-\\
&\quad \gamma I(x,y,z,t),\\
\frac{dR(x,y,z,t)}{dt} &= \gamma I(x,y,z,t).
\end{aligned}$$

where x, y, and z represent the strategy fractions, i.e. the number of individuals taking vaccination, IDM and both provisions at any time t, respectively.

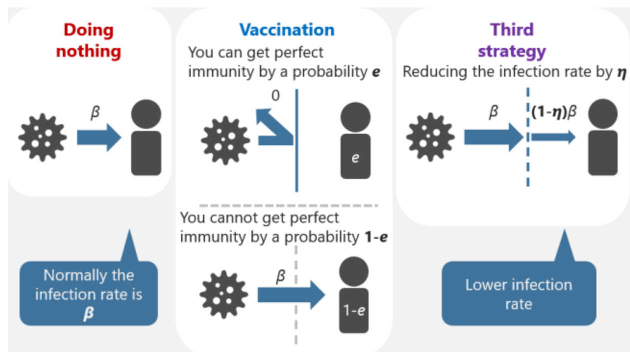

Figure 1. Schematics of the proposed model.

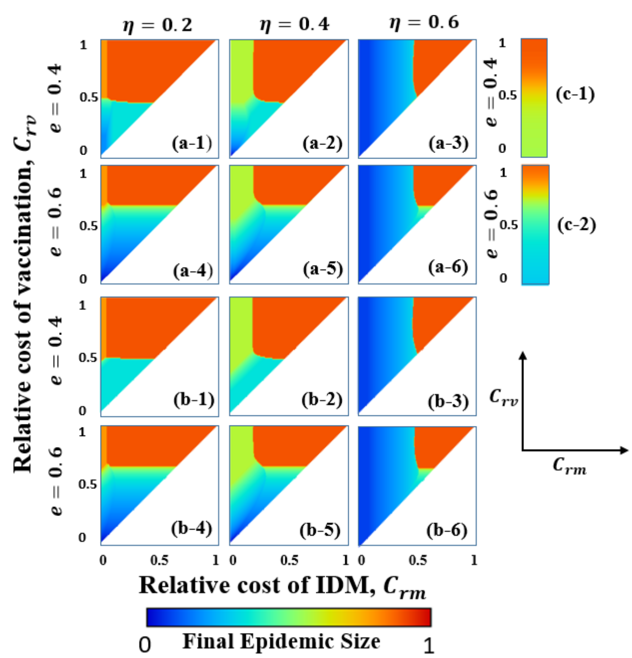

Figure 2. Final epidemic size (panels (a-*) for four-strategy model, panels (b-*) for three-strategy model, and panels (c-*) for two strategy model respectively).

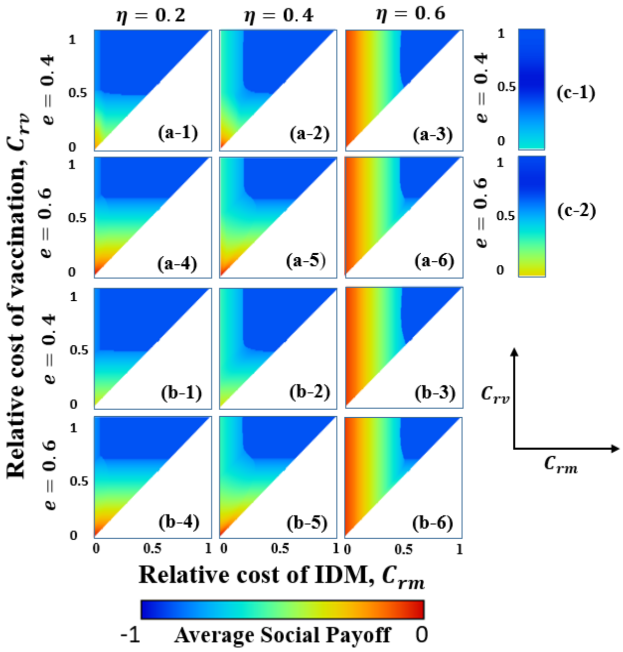

Figure 3. Average social payoff (panels (a-*) for four-strategy model, panels (b-*) for three-strategy model, and panels (c-*) for two strategy model respectively).

2.2 Payoff structure

Generally, an epidemic season continues until all infected individuals get fully recovered. During this course if a non-vaccinated individual gets infected must carry out the infection cost C_i. However, some extremely unlucky individuals who get infected despite taking either vaccination, IDM or both are charged with their respective provision cost as well as the infection cost. For convenience, the cost is rescaled by defining a relative cost of vaccination, namely $C_{rv} = C_v / C_i$ ($0 \leq C_v \leq 1$; $C_i = 1$). Likewise, the relative cost of IDM; C_{rm} and relative cost of combined strategy; C_{rb} are rescaled for the payoff structure. Eventually, at the end of an epidemic season, each individual's payoff is ensured by his/ her final state encapsulates in Table 2 whether committing to a particular provision or not as well as whether being healthy or infected.

2.3 Strategy updating

At the end of each season, individuals are allowed to alter or change their current strategy based on their payoffs in the previous epidemic season. Here, we use the strategy-based risk assessment (SB-RA) update rule to calculate the transition probabilities.

2.3.1 SB-RA update rule

Fukuda *et al.* [2] modified the imitation probability proposed by Fu *et al.* [4] that enables each individual to access the risk based on a socially averaged payoff resulting from adopting a certain strategy. The general form of the transition probability is given by:

$$P(S_i \leftarrow S_j) = \frac{1}{1+exp[-(\langle \pi_j \rangle - \pi_i)/\kappa]},$$

where, $\langle \pi_j \rangle$ is the average payoff obtained by averaging the collective payoff over individuals who adopt the same strategy as that of a randomly selected neighbor j of individual i. For our model we have twenty-four transition probabilities which can be summarized as:

$P(A \leftarrow B)$; $A \in \{HV, IV, \ldots, FFR\}$; $B \in \{V, M, B, FR\}$.

2.4 Evolutionary system equation

As each individual updates his strategy depending upon his last season's payoff following the SB-RA strategy update rule, therefore increasing or decreasing of x, y, and z is obvious.

$$\frac{dx}{dt} = -xy(1 + (1-e)exp[-R_0 R(x,y,z,\infty)])P(HV \leftarrow M)$$
$$-xz(1 + (1-e)exp[-R_0 R(x,y,z,\infty)])P(\mathrm{HV} \leftarrow \mathrm{B})$$
$$-x(1-x-y-z)(1 + (1-e)exp[-R_0 R(x,y,z,\infty)])P(HV \leftarrow D)$$
$$-xy(1-e)(1 - exp[-R_0 R(x,y,z,\infty)])P(\mathrm{IV} \leftarrow \mathrm{M})$$
$$-xz(1-e)(1 - exp[-R_0 R(x,y,z,\infty)])P(IV \leftarrow B) - x(1-x-y-z)(1-e)(1 - exp[-R_0 R(x,y,z,\infty)])P(\mathrm{IV} \leftarrow \mathrm{D})$$
$$-xz(1 + (1-e)exp[-(1-\eta)R_0 R(x,y,z,\infty)])P(HB \leftarrow V) + +xz(1-e)(1 - exp[-(1-\eta)R_0 R(x,y,z,\infty)])P(\mathrm{IB} \leftarrow \mathrm{V})$$
$$-xy\, exp[-(1-\eta)R_0 R(x,y,z,\infty)]P(HM \leftarrow V) + xy(1 - exp[-(1-\eta)R_0 R(x,y,z,\infty)])P(\mathrm{IM} \leftarrow \mathrm{V})$$
$$+x(1-x-y-z)exp[-R_0 R(x,y,z,\infty)]P(SFR \leftarrow V) + x(1-x-y-z)(1 - exp[-R_0 R(x,y,z,\infty)])P(\mathrm{FFR} \leftarrow \mathrm{V})$$

In a similar fashion we can also generate the dynamical equations for $\frac{dy}{dt}$ and $\frac{dz}{dt}$.

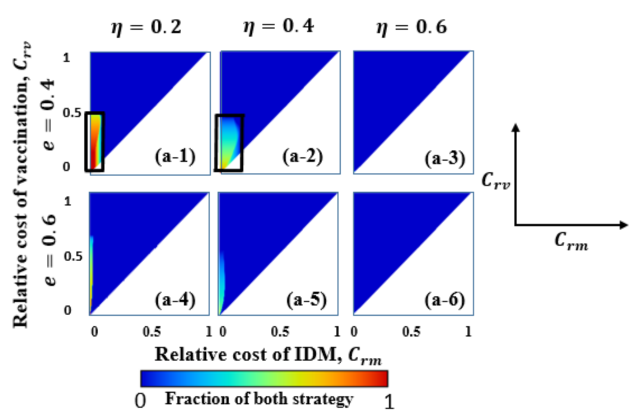

Figure 4. Fraction of population taking both (B) strategy.

3. Result discussion

Let us discuss the numerical results obtained from our theoretical model governed by the system of Equations (1).

Figure 2 provides the final epidemic size varying the parameter settings for different models namely, four-strategy (panels a-*), three-strategy (panels b-*), and two-strategy (panels, c-*). Similarly, Figure 3 indicates the average social payoff exactly for the same parameter settings and models. For the time being we limit our discussion by considering the effectiveness parameter $e = \{0.4, 0.6\}$ and the efficiency parameter $\eta = \{0.2, 0.4, 0.6\}$. Finally, Figure 4 shows the fraction of z, i.e. the strategy of taking both provisions at the same time.

Considering every parameter space continuously, a general tendency amid the entire population is observed when a large number of individuals plan to commit vaccination then there is a certain propensity of taking IDM (M) and both (B) in smaller number among the rest of the population and vice-versa. A further analysis reveals the fact that the final epidemic size is almost same for four-strategy model and three-strategy model irrespective to the parameters settings shown in Fig. 2. Despite having a very limited scope to survive, the fourth-strategy (B) can somehow improve the final epidemic size which is justified in panels (a-1 & a-2) of Figs. 2 & 4. On the other hand, the average social payoff is also relatively higher when the fourth-strategy (B) and third-strategy (M) is being introduced compared to that of two-strategy (V) case (depicted in Fig. 3). With respect to the fourth strategy (B) simultaneously taking both vaccination and IDM, let us point out several interesting discoveries as below. One thing is that the cases of the lowest reliability of vaccination ($e = 0.4$) only allows the fourth strategy surviving. Moreover, in the region of lower vaccination cost and extremely lower cost of IDM, a vast majority of people take the fourth strategy (see the solid line boxes in panels (a-1), and (a-2) in Fig. 4). It seems quite conceivable. This is because despite vaccination might be the first choice to avoid disease (even if he/ she is not able to acquire immunity, with some probability (e) he or she can acquire perfect immunity), lower e pushes individuals who committing vaccination to take IDM additionally. Taking both measures is fairly conceived as a more reliable alternative than taking one of those two.

Inevitably it can be said that the introduction of IDM and subsequently the fourth-strategy (B) seems meaningful and beneficial to the mass people during their rainy days as long as the both the relative costs considered are relatively cheaper as well as both the reliabilities are considerably higher.

4. Conclusion

In summary, with the evolution dynamics of epidemic spreading and strategy imitation, our model carefully explored the existence of third and fourth strategies in an infinite and well-mixed population which can be considered as an extension of the two-strategy model proposed by Kuga *et al*. [11]. Our theoretical results reveal the fact that introduction of these two new strategies seem beneficial to suppress the epidemic spreading from the cost effective point of view.

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