

# Time-dependent Queuing Approaches for Airport Immigration

Phuong Phi Doan<sup>1</sup>, Kazumichi Ohtsuka<sup>2,3</sup>, Katsuhiro Nishinari<sup>2</sup>

<sup>1</sup>Department of Aeronautics and Astronautics, School of Engineering, The University of Tokyo

<sup>2</sup>Research Center for Advanced Science & Technology, The University of Tokyo

<sup>3</sup>Department of Innovative Marketing Science, ECNEICS LABORATORY Co., Ltd.

## Abstract

Standard queuing theory describes only the long-term behavior of a queue. The passenger arrivals at the airport immigration fluctuate significantly during the day. The standard queuing theory cannot be used in this case for predicting the queue lengths. Instead time-dependent queuing approaches need to be applied. The queues and arrivals at the incoming immigration of Narita airport were observed on a single day. The utilization ratios show that the immigration was overloaded during a large part of the observation period. This condition requires the use of queuing approaches that can cope with overload. In this paper a deterministic fluid approximation and an approach based on the numerical integration of the Kolmogorov differential equations are discussed. Both approaches produce similar results and these results are reasonable compared to the observed queues.

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## 1 Introduction

Narita airport's role as an international transfer hub in Asia has been decreasing continuously because of capacity restrictions and emerging airport hubs in other metropolises [1] and since 2010 it has to compete with Haneda Airport for international flights. Despite the flight capacity restrictions foreign and Japanese passengers have complained about long queues at the immigration and the Japanese Justice Ministry is investigating how to improve the immigration procedures.

There are several options to reduce the queues at the immigration: reduce the service time, increase the number of open service counters and reduce arrival peaks. Optimizing the number of open service counters is similar to the staff-scheduling problem. Green et al [2] have reviewed queuing approaches for determining the required number of staff for service systems where the demand varies during the day. They discussed a time-dependent queuing approach to optimize the number of check-in staff at airports. Queuing theory was used by United Airlines to develop a computer system for scheduling staff at reservation offices and airports [3].

In this paper we explore which queuing approaches are appropriate for the incoming immigration of Narita airport for the foreign passengers. The queue lengths predicted by a deterministic fluid approximation and an approach based on the numerical integration of ordinary differential equations (ODE) are compared with actual measurement data.

## 2 Queuing Approaches

The standard queuing theory uses a constant arrival rate to describe the long-term behavior of a queue. However at the immigration the arrival rates can fluctuate significantly according to the flight schedule. Figure 1(a) shows the normalized arrival rates of foreign passengers for a 5 and 15-minutes time interval as observed at the incoming immigration of Narita airport. Time-dependent queuing approaches are necessary to analyze the airport immigration.

Green et al [2] have discussed time-dependent queuing approaches for three different cases: (a) systems with a high quality of service standard and a short service time (e.g. 5-10 minutes), (b) systems with a high quality of service standard and a medium to long service time (e.g. 1 hour)

and (c) overloaded systems where the quality of service standard is not so high.

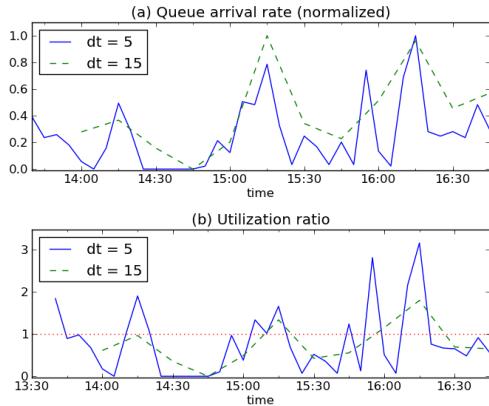


Figure 1: arrival rates (a) and utilization ratios (b) at the immigration of Narita airport

The immigration is obviously not case (b). Whether the immigration is an overloaded system depends on the utilization ratios during the day. The utilization ratio is the fraction of the service capacity that is used by the arriving passengers. The utilization ratio  $\rho$  is defined as

$$\rho = \frac{\lambda}{s\mu} = \frac{\lambda}{s} t_{service} \quad (1)$$

where  $\lambda$  is the arrival rate,  $s$  is the number of open service counters,  $\mu$  is the service rate of a single counter and  $t_{service}$  is the service time. A system is overloaded when the arrival rate is equal or larger than the total service capacity, i.e.  $\rho > 1$ . All variables were measured at the immigration of Narita airport and the resulting utilization ratios for a 5 and 15-minutes time interval are plotted in Figure 1(b). From the figure we can see that during a large part of the observation period the immigration was overloaded or nearly overloaded. The conclusion is that the immigration fits case (c). Two queuing approaches that can deal with overload will be discussed in the following sections.

## 2.1 Deterministic Fluid Approximation

Overloaded systems can be analyzed with a deterministic fluid approximation [4]. De Neufville [5] used this approach to analyze the flows and queues at airports. The deterministic fluid approximation assumes that during a time period the arrivals and departures from a queue occur at

evenly spaced intervals and that the service times are constant. If for each passenger the arrival time and departure time from the queue are recorded then the queue length at each moment can be determined by plotting the cumulative arrivals  $A(t)$  and cumulative departures  $D_q(t)$ .

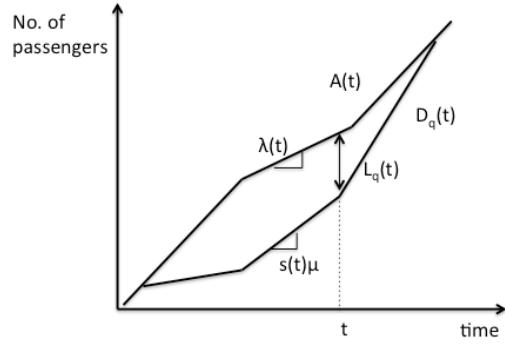


Figure 2: cumulative arrivals and departures

The queue length  $L_q(t)$  at each time is equal to the vertical distance between the two graphs as shown in Figure 2 and can be written as

$$L_q(t) = A(t) - D_q(t) \quad (2)$$

The cumulative diagrams can also be estimated with the arrival rate  $\lambda(t)$ , the number of open service counters  $s(t)$  and the service time  $t_{service} = 1/\mu$ .

## 2.2 Numerical Integration Approach

The deterministic fluid approximation can only estimate queues when a system is overloaded. If  $\lambda < s\mu$  the deterministic fluid approximation will never predict any queues although in reality queues are still formed because of fluctuations in the interarrival times and service times. The numerical integration approach can capture the queues in both overloaded and non-overloaded situations. The method consists of deriving the ordinary differential equations of the time-dependent queuing theory and solving the equations numerically. The numerical integration approach has been applied to analyze the takeoff and landing queues of aircraft [5], to estimate the optimal schedule of police patrol cars [6] and to assist managers with the allocation of firefighting helicopters among various bases in a region [7].

A detailed discussion of the time-dependent queuing theory can be found in [8]. For the

immigration we assume that passengers arrive randomly with an average rate  $\lambda(t)$  and an exponential interarrival time distribution. The service time distribution at the service counters is also assumed to be exponential. The expected service time is equal to  $1/\mu$ . The number of open service counters  $s(t)$  is time-dependent. The immigration with these arrival and service characteristics is an M/M/ $s_t$  queuing system that can be modeled as a birth-death process. Let the stochastic variable  $X(t)$  represent the total number of passengers in the queue and at the service counters at time  $t$  and let  $p_j(t)$  denote the transition probability that there are  $j$  passengers in the system after time  $t$  has passed. If there are initially  $k$  passengers in the system then  $p_k(0) = 1$  and the transition probability  $p_j(0)$  for  $j \neq k$  equals zero. There is a capacity limit  $m$  to the number of passengers at the immigration. The evolution of the transition probabilities can be obtained from a flow balance. Figure 3 shows the rate transition diagram with the flows into and out of every possible state of  $X(t)$ . The change in the state of the system  $dp_j(t)/dt$  is equal to difference in the flow rates. Application of the flow balance results in the Kolmogorov differential equations for the M/M/ $s_t$  queuing system [6]

$$\begin{aligned} \frac{dp_0(t)}{dt} &= -\lambda(t)p_0(t) + \mu p_1(t) \\ \frac{dp_j(t)}{dt} &= \lambda(t)p_{j-1}(t) - [\lambda(t) + j\mu]p_j(t) \\ &\quad + (j+1)\mu p_{j+1}(t) \\ &\quad \text{if } 1 \leq j < s(t) \\ \frac{dp_j(t)}{dt} &= \lambda(t)p_{j-1}(t) - [\lambda(t) + s(t)\mu]p_j(t) \\ &\quad + s(t)\mu p_{j+1}(t) \\ &\quad \text{if } s(t) \leq j < m \\ \frac{dp_m(t)}{dt} &= \lambda(t)p_{m-1}(t) - s(t)\mu p_m(t) \end{aligned} \quad (3)$$

This is a system of ordinary differential equations that can be solved numerically. The expected number of passengers in queue can then be obtained as

$$L_q(t) = \sum_{j=s(t)+1}^m [j - s(t)]p_j(t) \quad (4)$$

The input variables for the numerical integration approach are the arrival rate  $\lambda(t)$ , the number of open service counters  $s(t)$ , the service rate  $\mu = 1/t_{service}$  and the system capacity  $m$  which can be determined from the observation data.

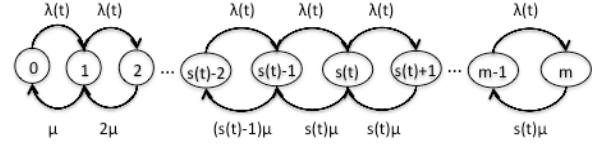


Figure 3: rate transition diagram for an M/M/ $s_t$  queuing system with capacity  $m$

### 3 Results

On February 6<sup>th</sup> 2011, a day for which a large number of incoming foreign passengers were expected, the queues were observed at the immigration of Narita airport. The arrival time of each passenger was recorded and the number of open service counters was observed for every three minutes. Videos were recorded of the queues and passengers at the service counters. From the videos the queue lengths for every 5 minutes were determined and the service times of 105 random foreign passengers were measured.

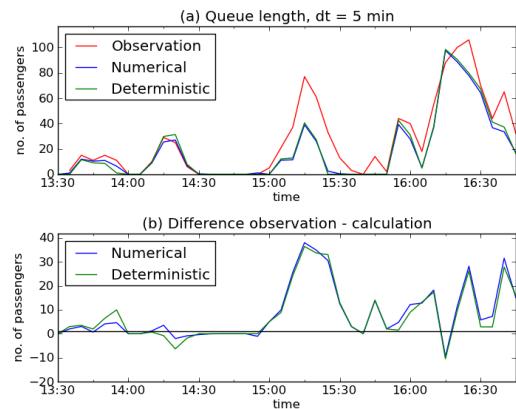


Figure 4: queue lengths at the foreign service counters

The results of the deterministic fluid approximation and the numerical integration approach are plotted in Figure 4(a) together with the observed queue lengths at the foreign service counters. The input data for both approaches is the same and a 5-minute time interval is used. Both approaches produce almost the same results because the immigration is overloaded during a

large part of the observed period. Figure 4(b) shows the differences in the number of passengers between the observation and the calculated queue lengths. Compared to the observed values both approaches produce reasonable results except between 15:00 and 15:30 and at the end of the period when the observed queues are longer than calculated. Calculations for a 1-minute time interval give differences of the same order as for a 5-minute time interval.

## 4 Conclusion

In this paper we explored suitable queuing approaches for the foreign service counters at the immigration of Narita airport. The immigration was observed for 3 hours consecutively on one day. The arrival rates changed significantly during the observation period and the utilization ratios were high. Under these circumstances the deterministic fluid approximation and the numerical integration approach are good choices. However the immigration was only observed for one day. On other days the utilization ratios might be lower although we can still expect peaks in the arrival rates because passengers from a flight arrive in batches at the immigration. The two approaches should be tested for different passenger demands.

Although the approaches produce reasonable results there are some periods where the calculations differ from the observations. Several possible causes can be identified. First during peak periods the queues in front of the service counters made it difficult to determine the number of open service counters. Second the service times were assumed to be equal for all foreign passengers. According to the manager of the immigration office the average service time for foreigners from Asian countries is longer than from the non-Asian countries. Third the number of passengers in the queue was counted from the recorded videos. These videos were recorded with Standard-definition video quality, which made it difficult to distinguish the waiting passengers on the computer screen during peak periods. Future research of the immigration will address these issues and it will focus on determining the optimal number of staff at the service counters.

## 5 Acknowledgement

The authors would like to thank Narita airport for the support with gathering observation data.

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